Quarks, Gluons, and Strings

Barak Bringoltz

IIAR – the Israeli Institute for Advanced Research

Thanks to many friends and colleagues:

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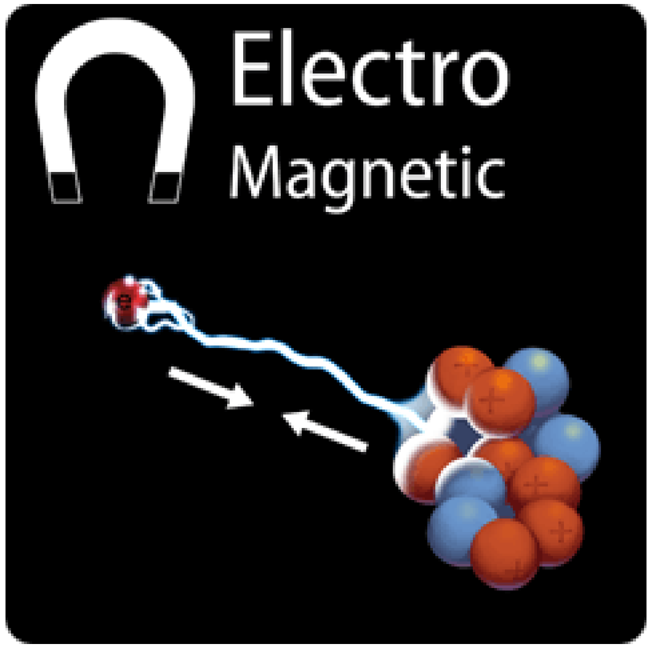
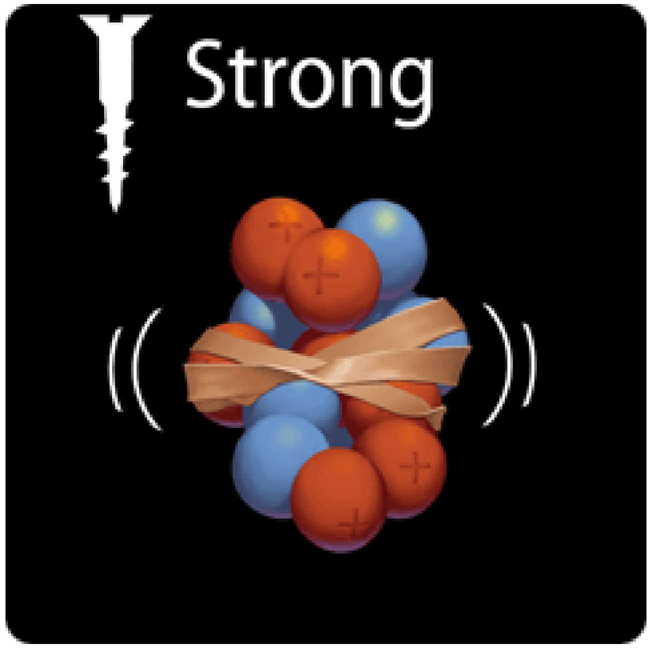
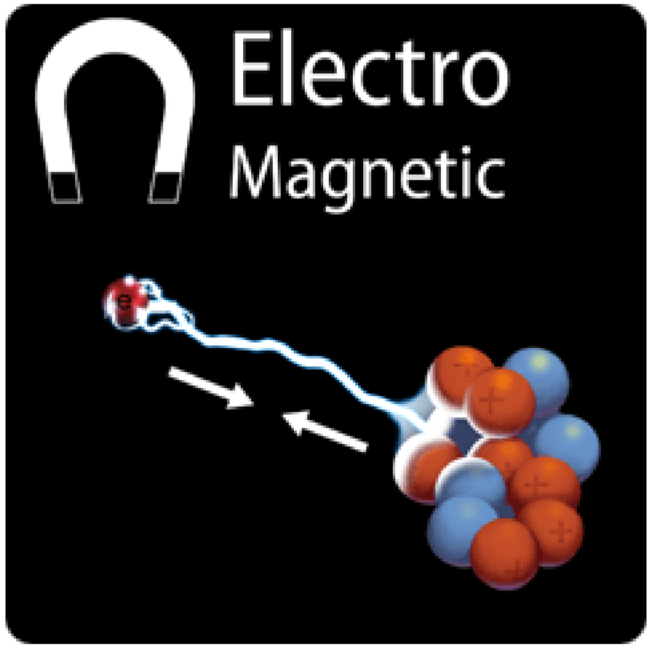
Four Fundamental Forces in nature: the standard model of particle physics

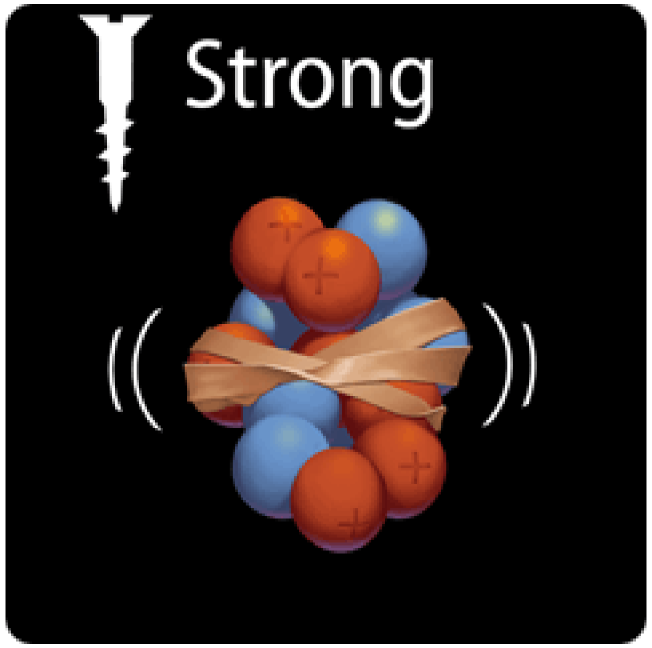




This is the force I work on

Four Fundamental Forces in nature: (pre possible LHC discoveries!)

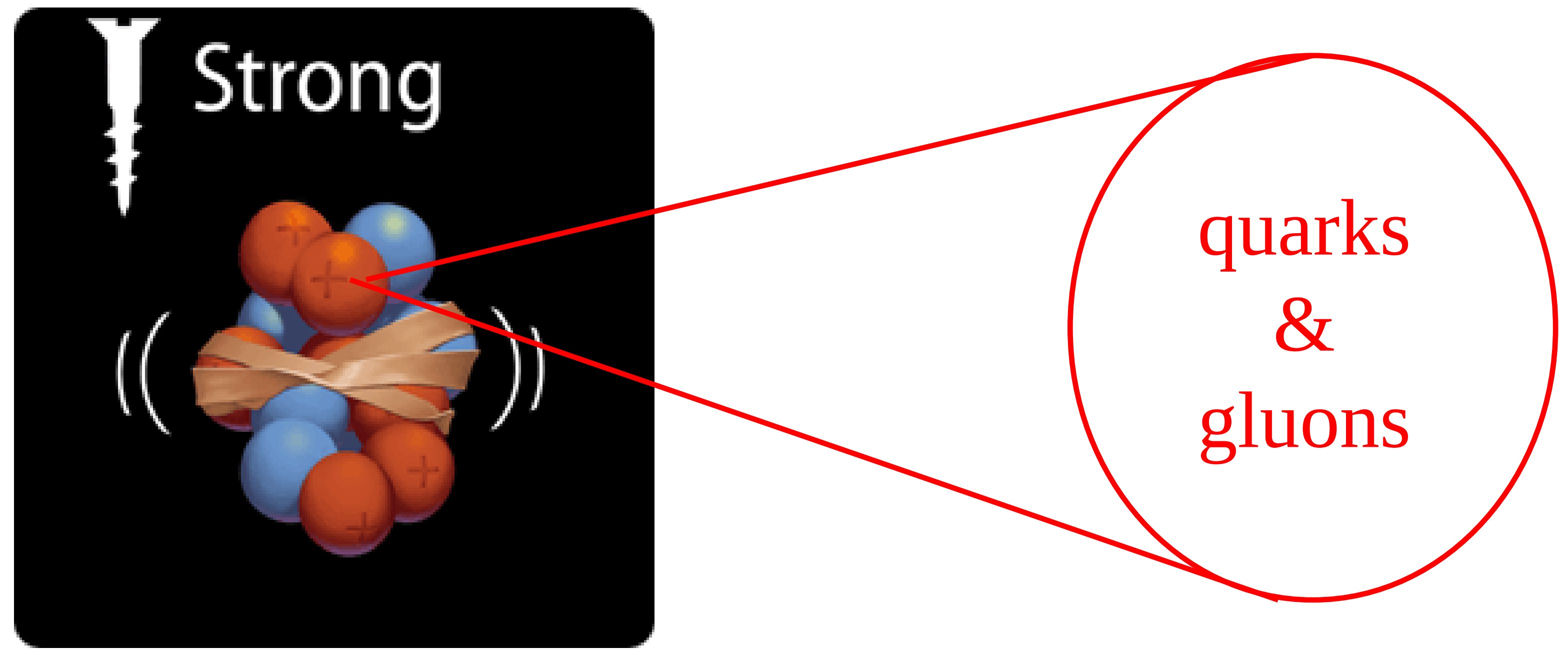




This is the force I work on

As picture “shows”: strong force binds protons & neutrons into nuclei

Red spheres are protons
Blue spheres are neutrons



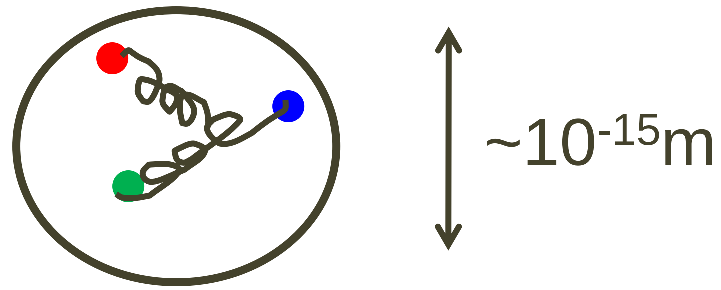
- These forces are only remnants of stronger forces .
- Stronger forces describes how fundamental particles called “quarks” & “gluons” interact and combine to create bound states that we call nuclear particles

Physics of the strong forces of quark & gluons underlies nuclear physics

Two type of nuclear particles (commonly called “hadrons”)

Nuclear particles made out of three quarks each: “Baryons”

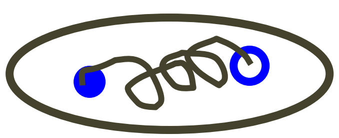
protons, neutrons, …



$M_{proton} \sim 1000$MeV
$M_{quark} \sim 1\text{-}5$ MeV

Nuclear particles made out of pairs of quarks and anti-quarks: “mesons”

pi-mesons, rho-mesons, …



These are the particles that nuclear physicists deal with:
they collide with each other, interact with each other, form nuclei, decay radioactively, etc.

So where are the gluons ?
Why is nuclear physics about bound states of quarks and not quarks ?

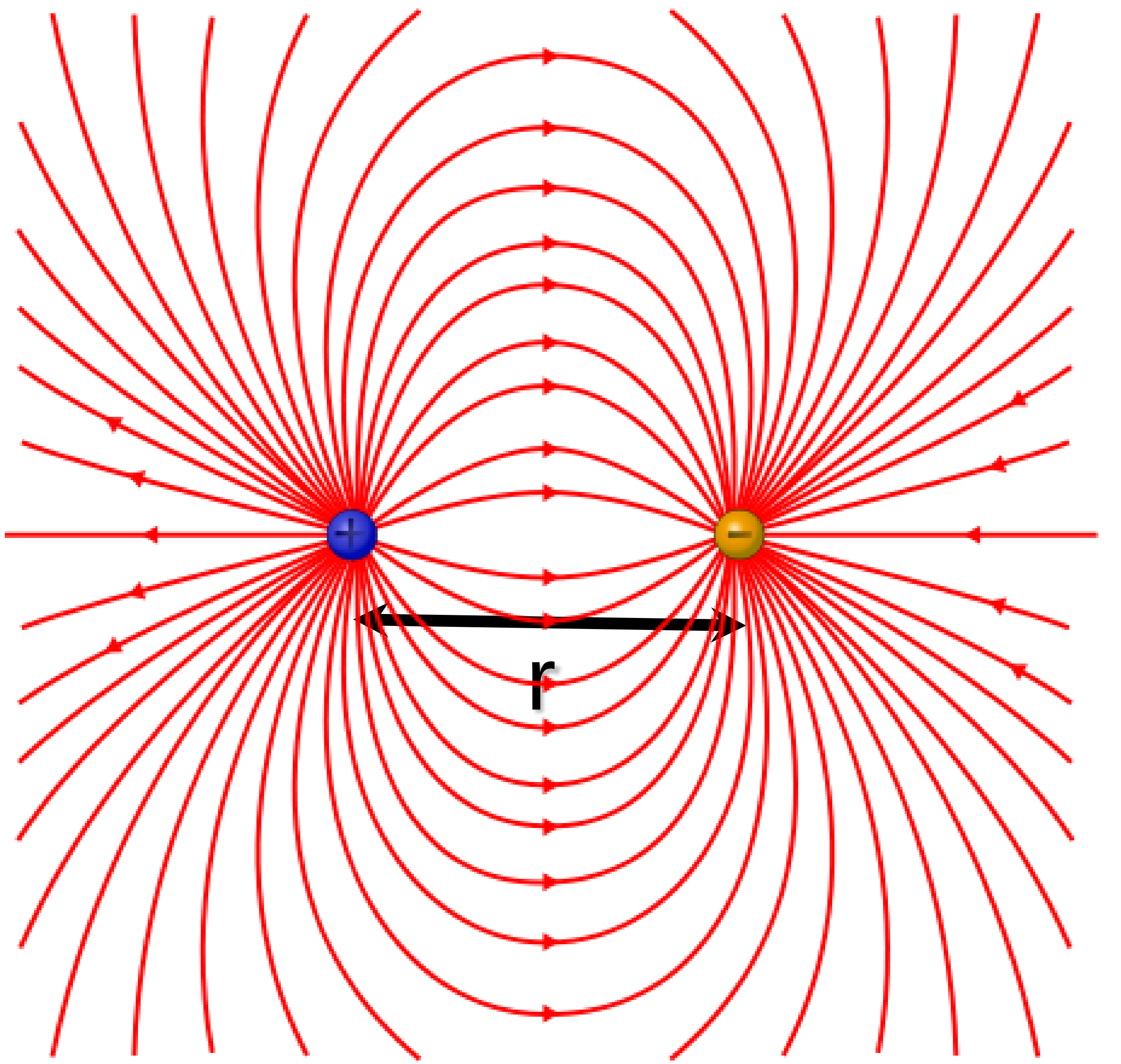
The strong force

Analogy between the strong force & electro-magnetism

Electro-magnetism (EM): forces between electrically charged particles

→ a property called "electric charge" determines how EM fields affect particles
→ theory describes electrons, positrons & electro-magnetic fields (or EM radiation)

Quantum Electro-magnetism *(aka Quantum Electro-Dynamics; or QED)* describes electrons, positrons & photons

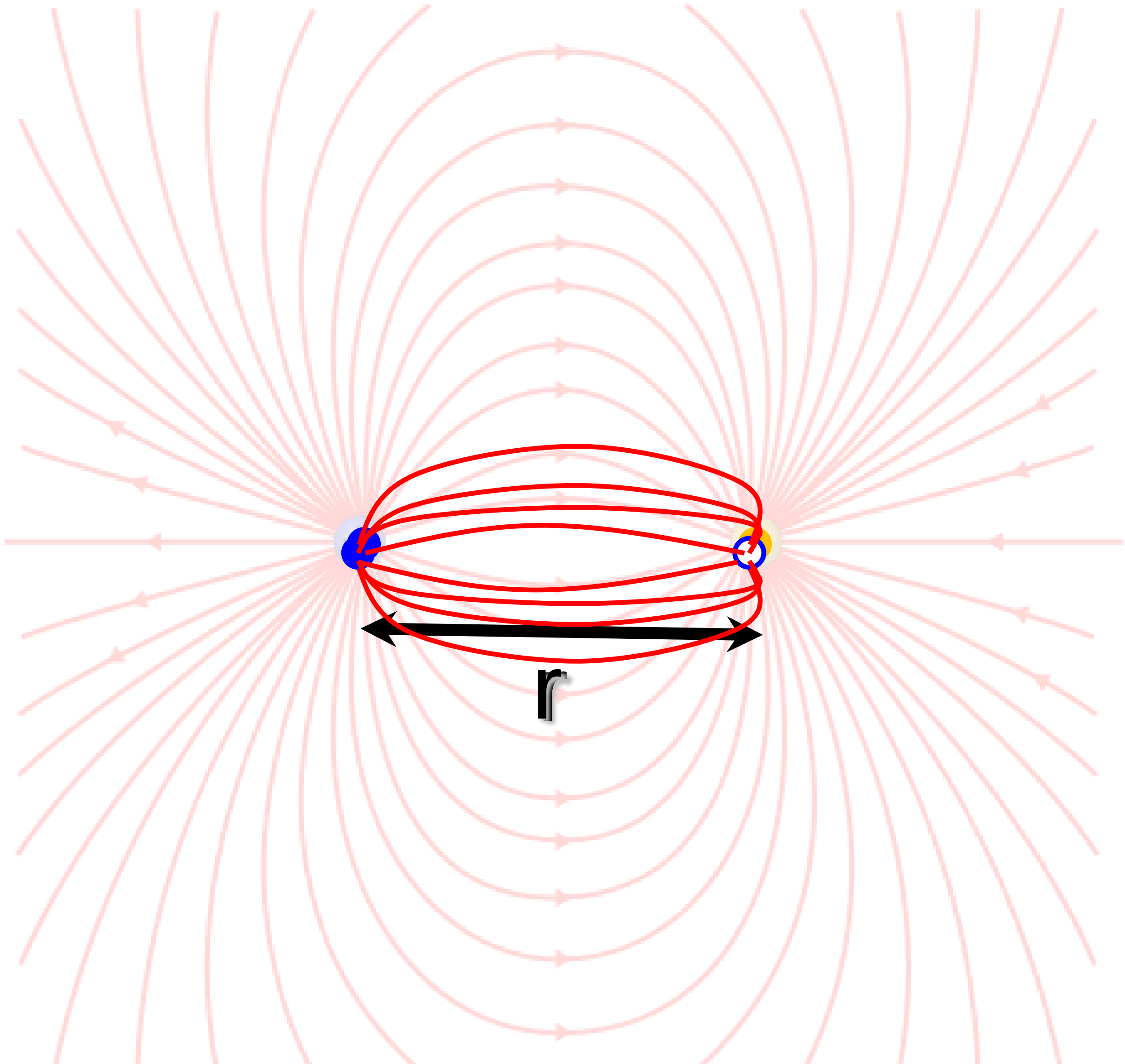


Analogy between strong force & electro-magnetism

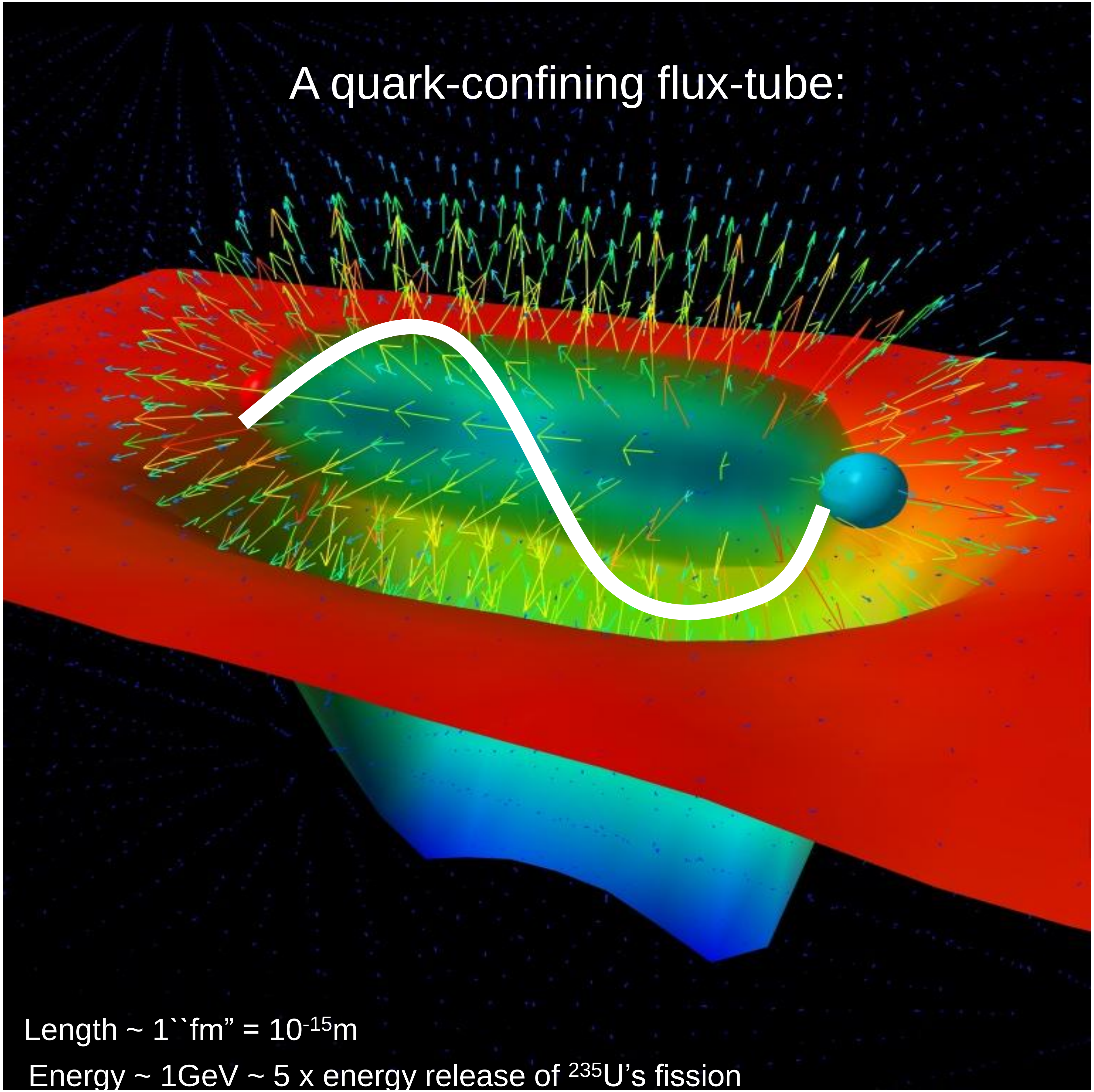
Strong forces: forces between "colored" charged particles

→ a property called "color charge" determines how strong forces affect particles
→ theory describes quarks, anti-quarks & "chromo" fields/"chromo" radiation

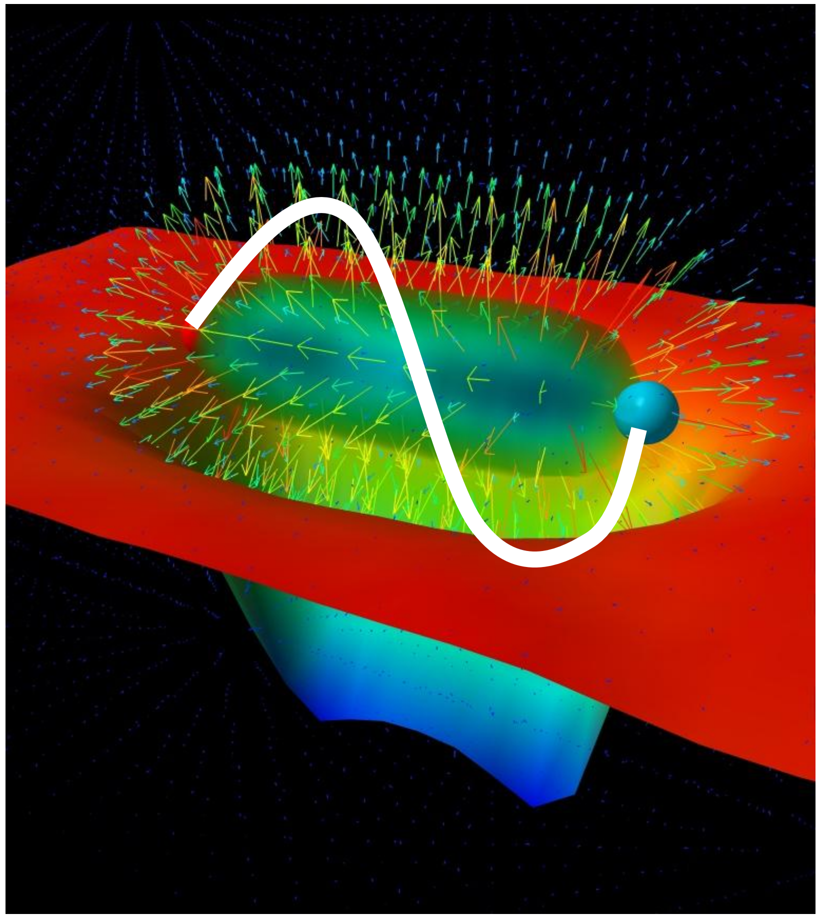
Quantum Chromo Electro-magnetism *(aka Quantum Chromo-Dynamics; or QCD)* describes quarks, anti-quarks & gluons



From F. Wilczek's Nobel lecture 2004, Stockholm



From F. Wilczek's Nobel lecture 2004, Stockholm



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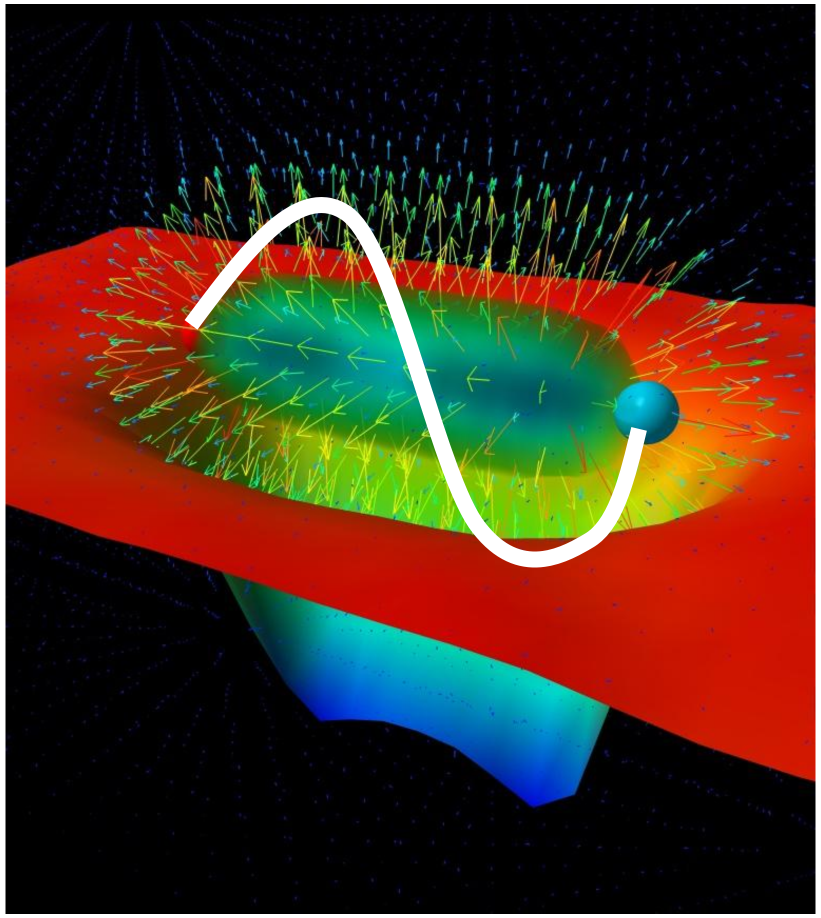
Study of flux-tubes

- **Why?** Everything you see around you is made up of quarks confined by flux-tubes (protons, neutrons, mesons, …)

- **How?** Use computer simulations
    - Access to phenomena that are hard/impossible to access experimentally.
    - Provide benchmarks for analytic approaches to the physics of quarks and gluons

From F. Wilczek's Nobel lecture 2004, Stockholm



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- Described by Quantum Chromodynamics (QCD)

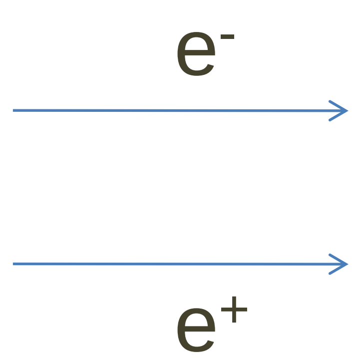
  - QCD forces are very strong ! 100 times stronger than electric force.

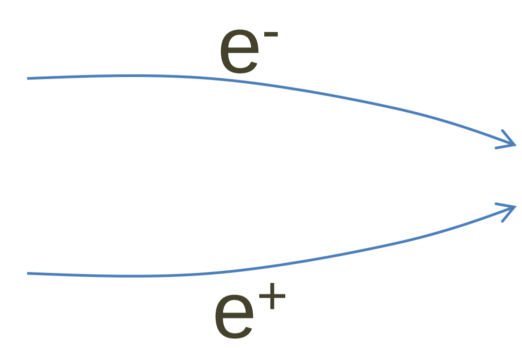
→ **Cannot use perturbation theory !**

Electro-Magnetism (or QED): interaction is weak

Interaction = small perturbation over an “idealized” non-interacting world

pictorially: Let two electrons propagate

Importance(

) >> Importance(

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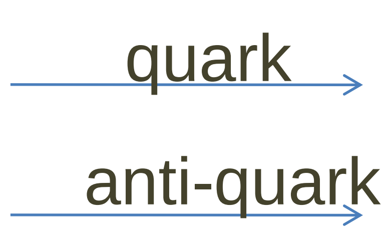
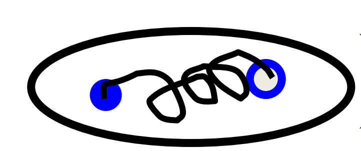
$\Longrightarrow$ predictions = analytic expansions in the interaction strength

Strong interactions (QCD): interaction is very strong

While QCD describes quarks, in nature we only see their bound states

Importance(

) << Importance()

$\Longrightarrow$ Perturbative approach has very limited abilities within QCD

Outline

Part I: How we study quarks & gluons non-perturbatively

(the lattice approach)

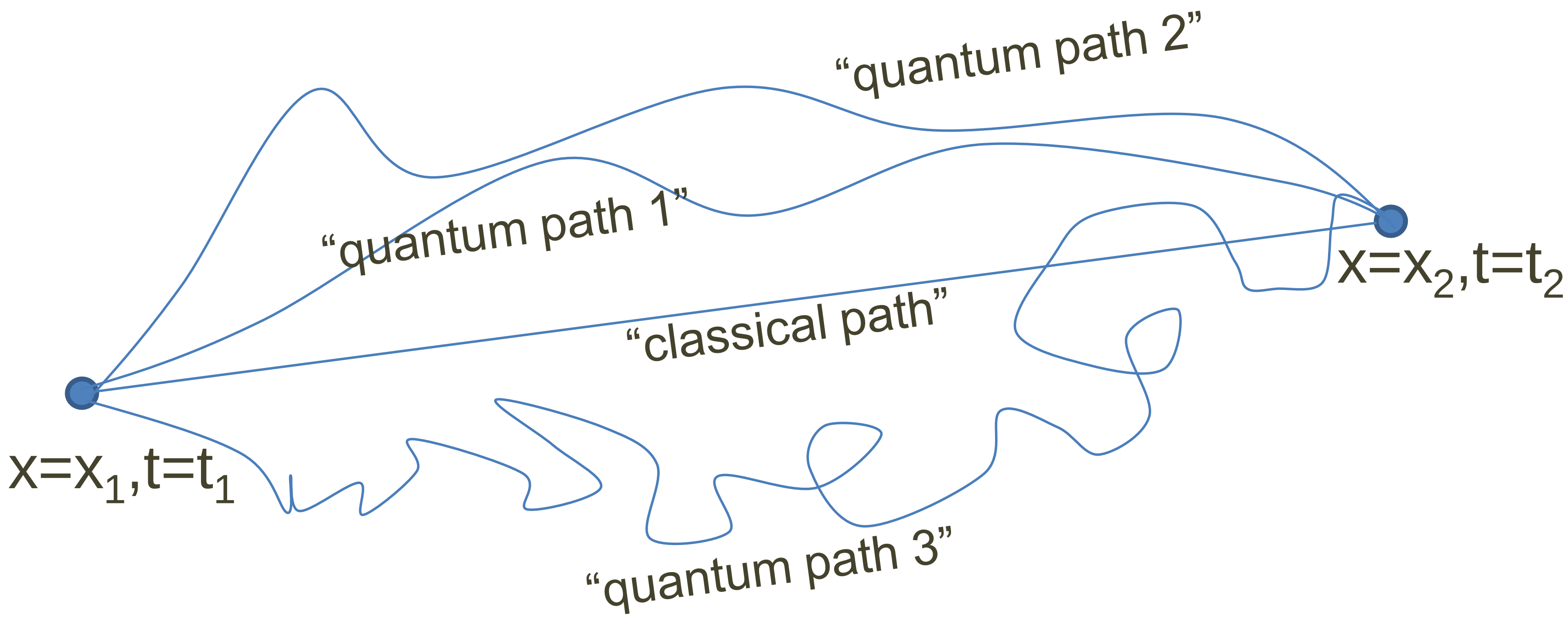
Part II: Examine QCD Flux-tubes as strings

Part I:

How we study quarks & gluons non-perturbatively

Use “path integral” approach to Quantum Mechanics

- In QM physical processes (say particle travels from x_1 to x_2) are stochastic.
- A quantum process is described by a weighted “sum” over all possible realizations.



Stochastic variable is spatial position of particle

Physical observable f(x(t)) determined by a prescribed function P(path)

(e.g. average position, average velocity, …)

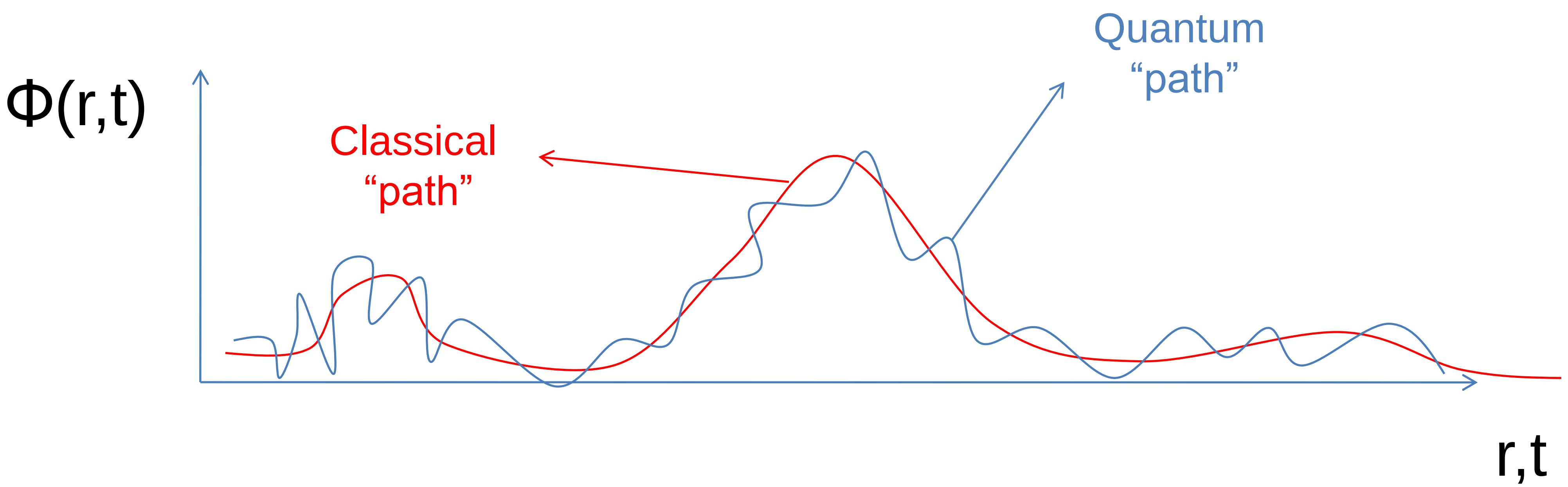
$$\langle f(x)\rangle = \frac{1}{Z}\sum_{\text{path}} f(x(\text{path}))P(\text{path}) \,, \qquad Z = \sum_{\text{paths}} P(\text{path})$$

$\Longrightarrow$ Interpret P as probability

In our case:

- Quantum degrees of freedom (the stochastic variables) represent:
  - Strength of chromo-electro-magnetic radiation → ~ density of gluons
  - Density of quarks and anti-quarks

Our stochastic variables are functions in space (“fields”) $\phi(\vec{r},t)$



$$P(\text{path}) \rightarrow P(\phi(\vec{r},t))$$

Theoretical framework called “Quantum Field Theory” (QFT)

In our case:

- Quantum degrees of freedom (the stochastic variables) represent:
  - Strength of chromo-electro-magnetic radiation → ~ density of gluons
  - Density of quarks and anti-quarks

Our stochastic variables are functions in space ("fields")

$$\phi(\vec{r},t)$$

$$P(\text{path}) \to P(\phi(\vec{r},t))$$

- Contact with experiment:

P(path)

$$\langle f(\phi)\rangle = \frac{\int \; \prod_{\vec{r},t} d\phi(\vec{r},t)\; P\big(\phi(\vec{r},t)\big)\; f(\phi)}{\int \; \prod_{\vec{r},t} d\phi(\vec{r},t)\; P(\phi(\vec{r},t))}$$

Can calculate:
energy, pressure, density, or observables that probe particle masses, etc.

- Integrals not well defined: (infinite number of dof's).

  Can overcome this difficulty in perturbations theory, but that is not useful for us !

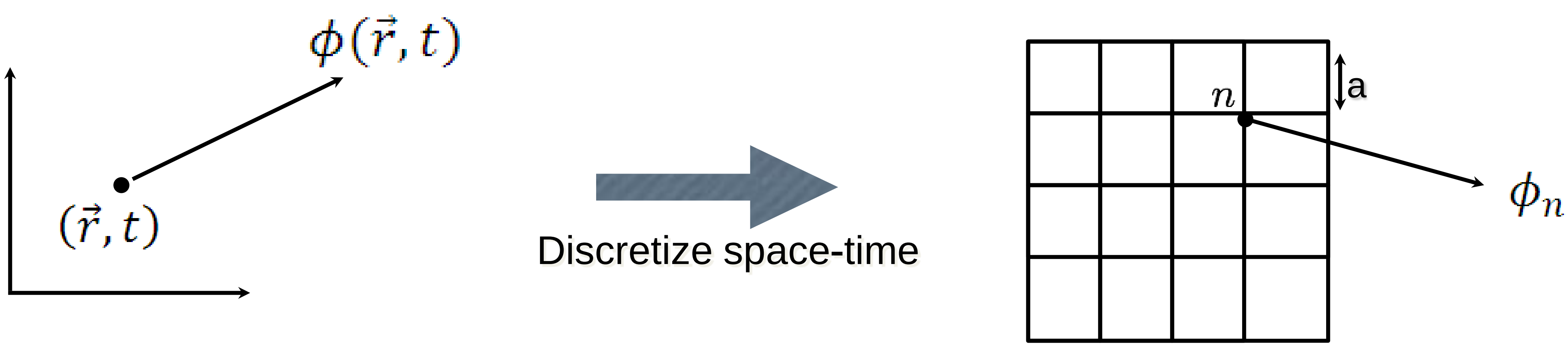
So how do we approach QCD non-perturbatively ?

PHYSICAL REVIEW D VOLUME 10, NUMBER 8 15 OCTOBER 1974

Confinement of quarks*

Kenneth G. Wilson
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850
(Received 12 June 1974)

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has a computable strong-coupling limit; in this limit the binding mechanism applies and there are no free quarks. There is unfortunately no Lorentz (or Euclidean) invariance in the strong-coupling limit. The strong-coupling expansion involves sums over all quark paths and sums over all surfaces (on the lattice) joining quark paths. This structure is reminiscent of relativistic string models of hadrons.



Path integral is **well-defined & finite** on lattice $\Rightarrow$ can do many things, not just perturbation theory

- Analytic non-perturbative methods (e.g. expansion in 1/[interaction strength])
- Perform the integral via with computer simulations

Wilson’s paper = birth of lattice QCD

- Today work mostly (but not exclusively) involves computer simulations:




- Computer simulations are of the Monte Carlo type

1. Generate a list of snapshots of the fields on the lattice weighted by

$$P\big(\phi(\vec{r},t)\big) \quad \Longrightarrow \quad \phi^{(1)},\phi^{(2)},\ldots,\phi^{(\mathcal{N})}$$

2. Measure “observables”: $$\langle f(\phi)\rangle = \frac{1}{\mathcal{N}}\sum_{i=1}^{\mathcal{N}} f\left(\phi^{(i)}\right) + \text{"}\frac{1}{\sqrt{\mathcal{N}}}\text{"}$$

Wilson's paper = birth of lattice QCD

- Today work mostly (but not exclusively) involves computer simulations:

Have a set of stochastic fields defined on a lattice

&

governed by some probability density:

Very similar to:

✓ Statistical mechanics in condensed matter.

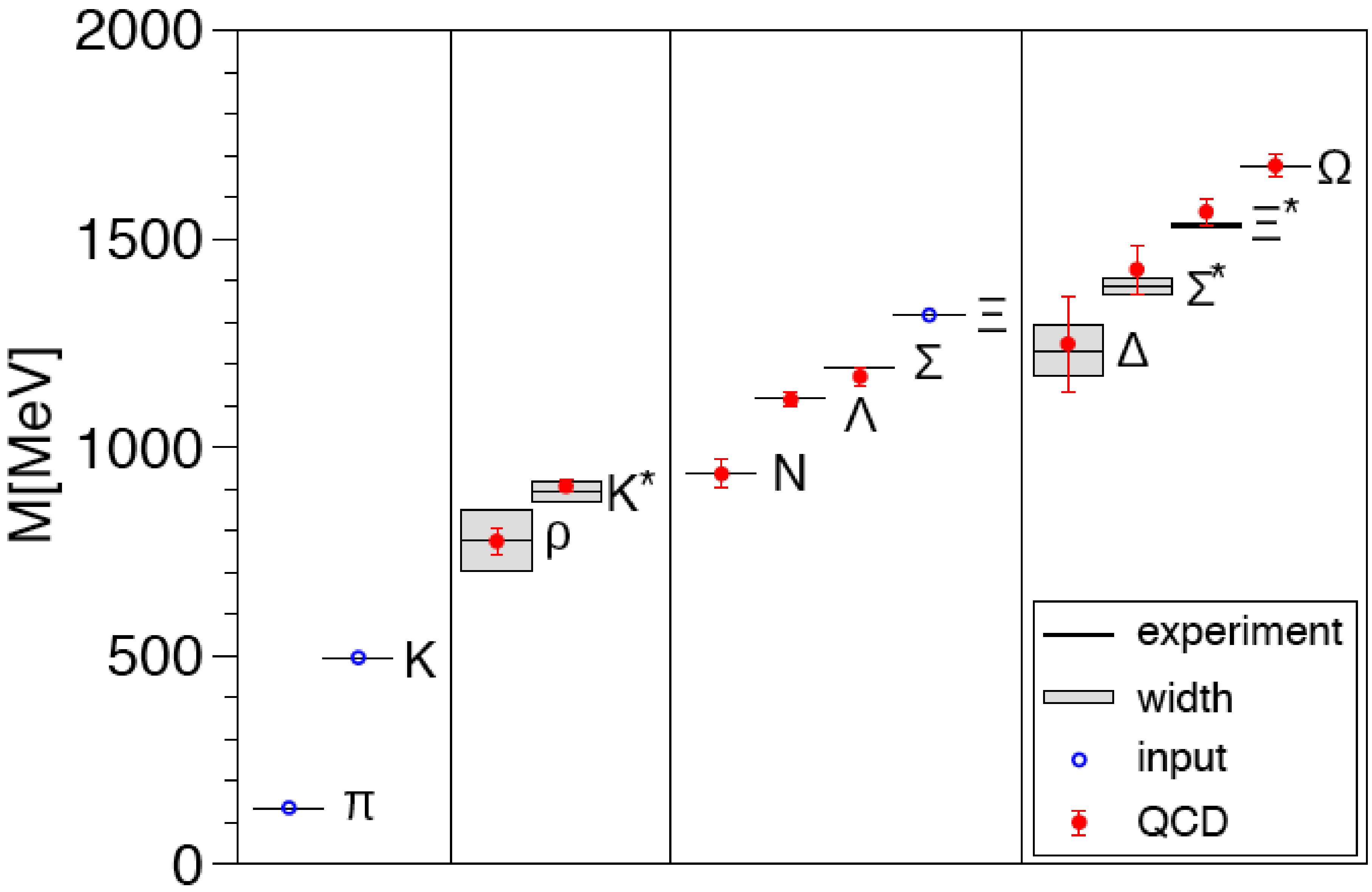
✓ Markov Random Fields in machine learning / computer vision.

$i=1$

3. Take continuum limit.

Field has come a long way since 1974 !

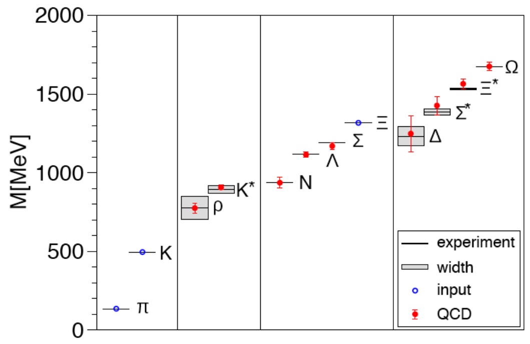
For example:



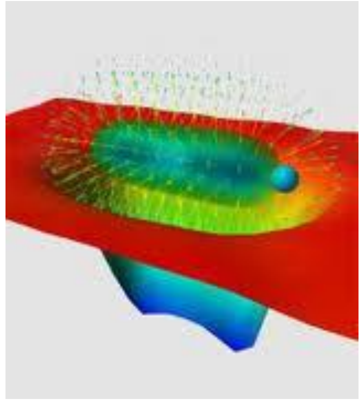
Budapest-Marseille-Wuppertal Collaboration
Science 322:1224-1227,2008.

Lattice community is busy with many aspects of QCD
(& other strongly coupled theories !)

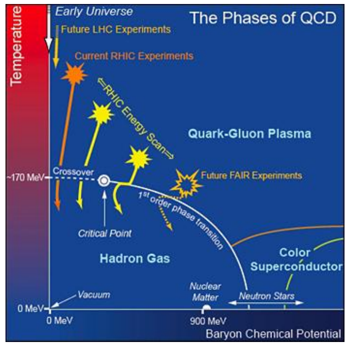
- Spectrum calculations



- Confining flux-tubes



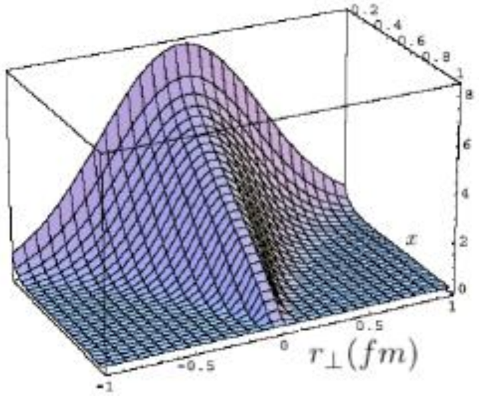
- High Temperature & density



- Generalizations of QCD

Lower dimensions,
large number of colors,
effective QCD models, ...

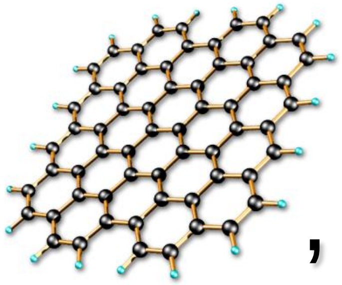
- Properties of nuclear particles



- Other strongly-coupled physics


LHC

,


Graphene

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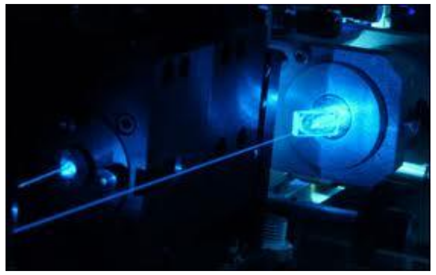

Ultra-cold atoms

Part II:

Examine QCD Flux-tubes

Look at their energy spectrum

,,

So how do you calculate *particles* energies in lattice QCD ?

- Consider the following wave functions at time = 0 and at time = t

For pure quantum states: $$G(t) = \Psi_\alpha^*(0) \cdot \Psi_\alpha(t) = e^{-E_\alpha t}$$

Wave function of quantum state α at time t=0

Wave function of quantum state α at time t

- Can express G(t) as a path integral average

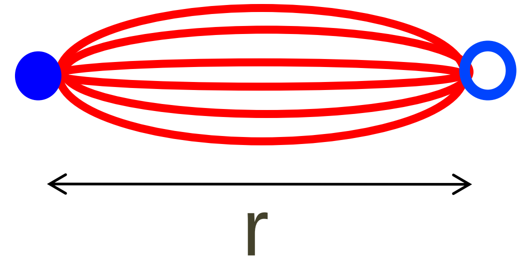
$$G(t) = \langle f_\alpha^*\left(\phi(0)\right) f_\alpha(\phi(t))\rangle$$

- Can calculate G(t) with Monte-Carlo

$$G(t) = \langle f_\alpha^*\left(\phi(0)\right) f_\alpha(\phi(t))\rangle = \frac{1}{\mathcal{N}} \sum_{i=1}^{\mathcal{N}} f_\alpha^*\left(\phi^{(i)}(0)\right) f_\alpha \left(\phi^{(i)}(t)\right) + \text{``}\frac{1}{\sqrt{\mathcal{N}}}\text{''}$$

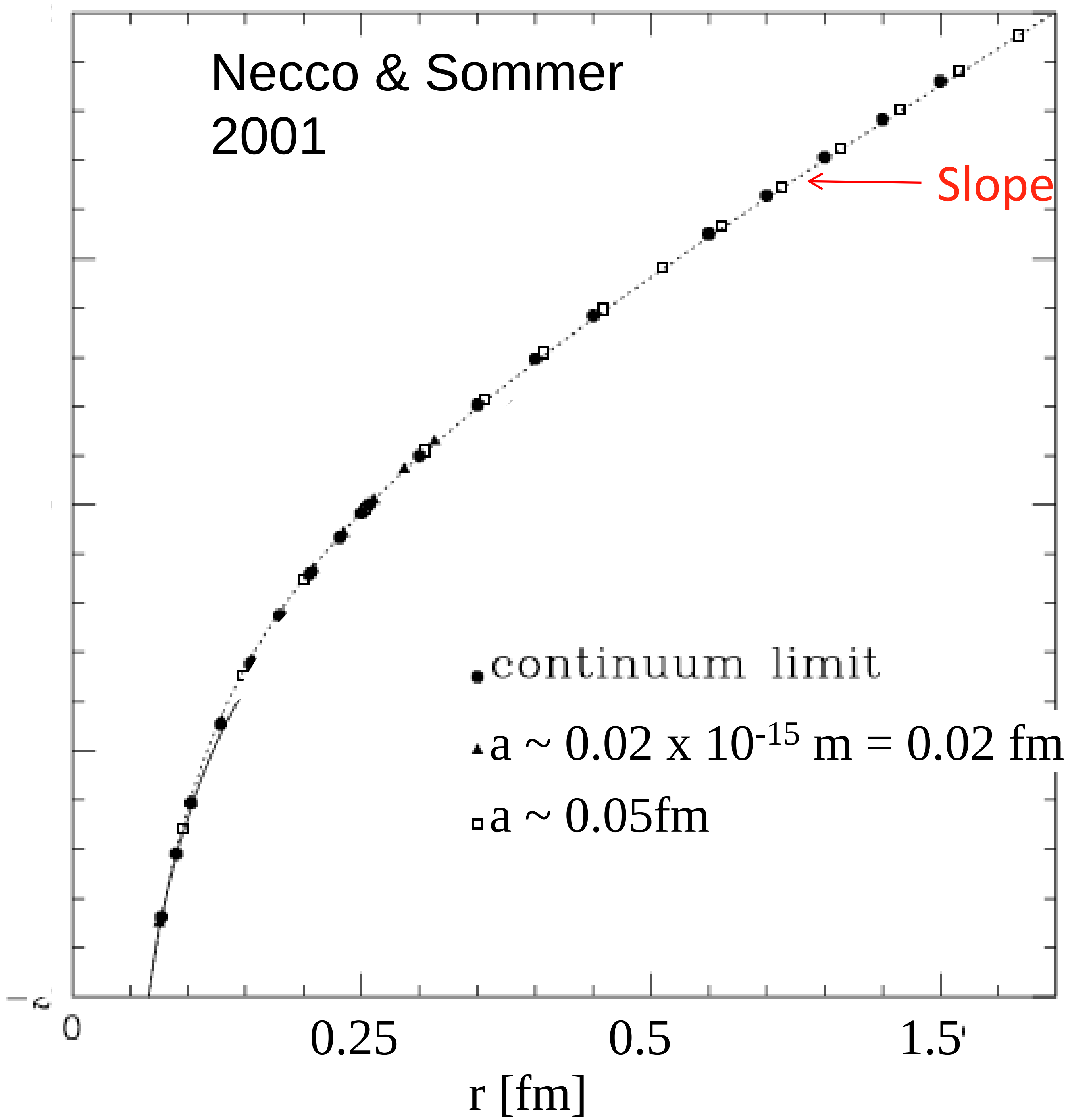
- Extract E_α from fit

If wave function ~



then $E_0(r) = V_{\text{quark-antiquark}}(r)$

V(r)



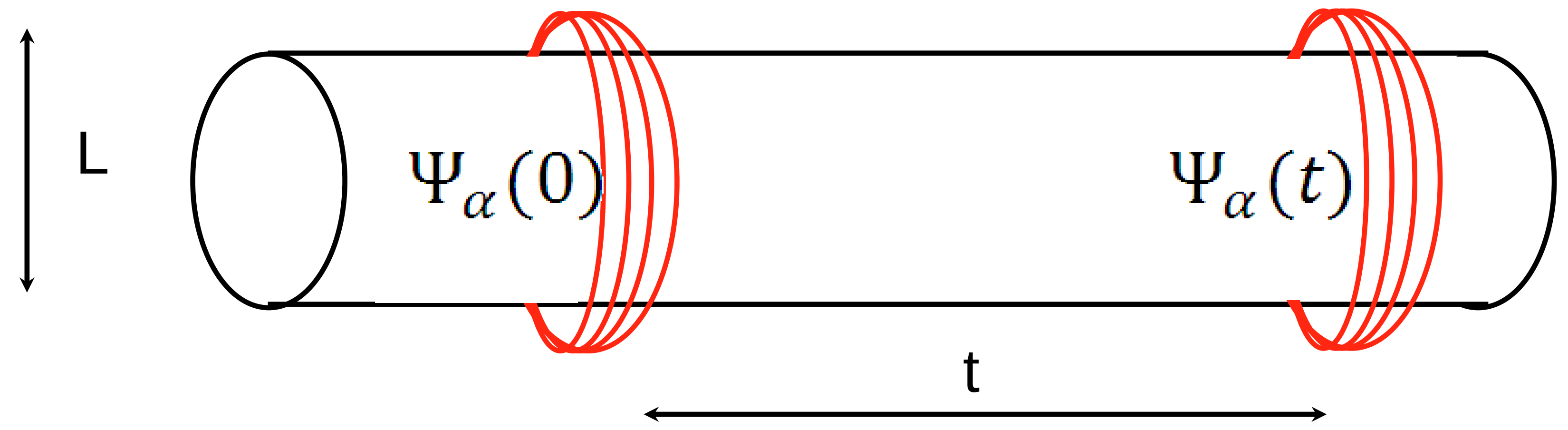
Slope = σ = energy per unit length

V(r) ~ **σ**r at large r:
"Linear confinement"

Remainder of talk: spectra of closed flux-tubes

Athenodorou, BB, Creutz, del Debbio, de Forcrand, Kuti, Meyer, Michael, Necco, Lucini, Lotini, Panagopolous, Ohta, Rossi, Schierholz, Teper, Vicari, Wenger, Wingate, ...

Use wave functions that describe fluxes that wrap around a spatial cylinder

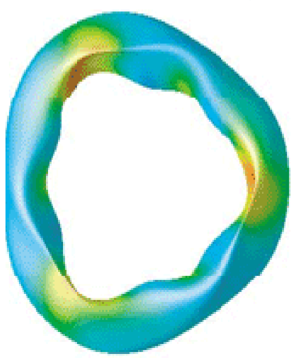
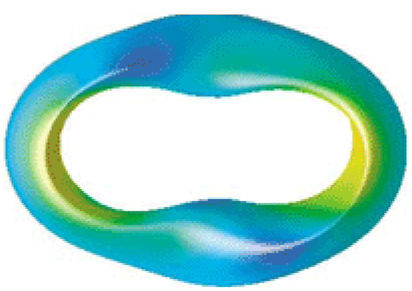


For pure quantum states: $G(t) = \Psi^*_\alpha(0) \cdot \Psi_\alpha(t) = e^{-E_\alpha t}$

Use the state-of-the-art "variational method" (PCA in machine learning & computer vision)

- Engineer Ψ_α such that it corresponds to a pure quantum state

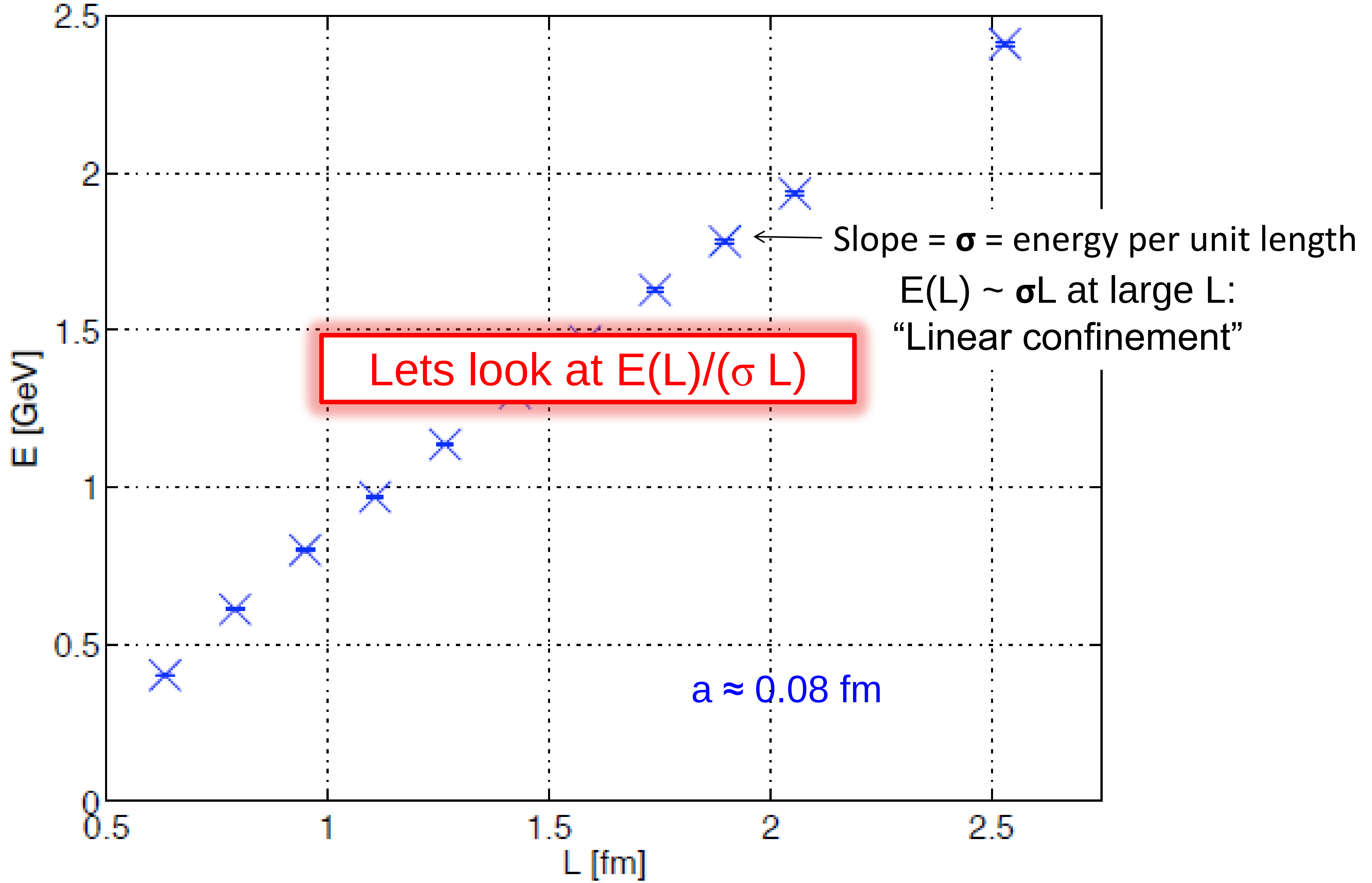
- Search for pure quantum state in a large basis: $\varphi_1, \varphi_2, \varphi_3, \ldots, \varphi_M$

$\Psi_\alpha = a_1 \cdot$  $+a_2 \cdot$  $+a_3 \cdot$  $+\ldots$

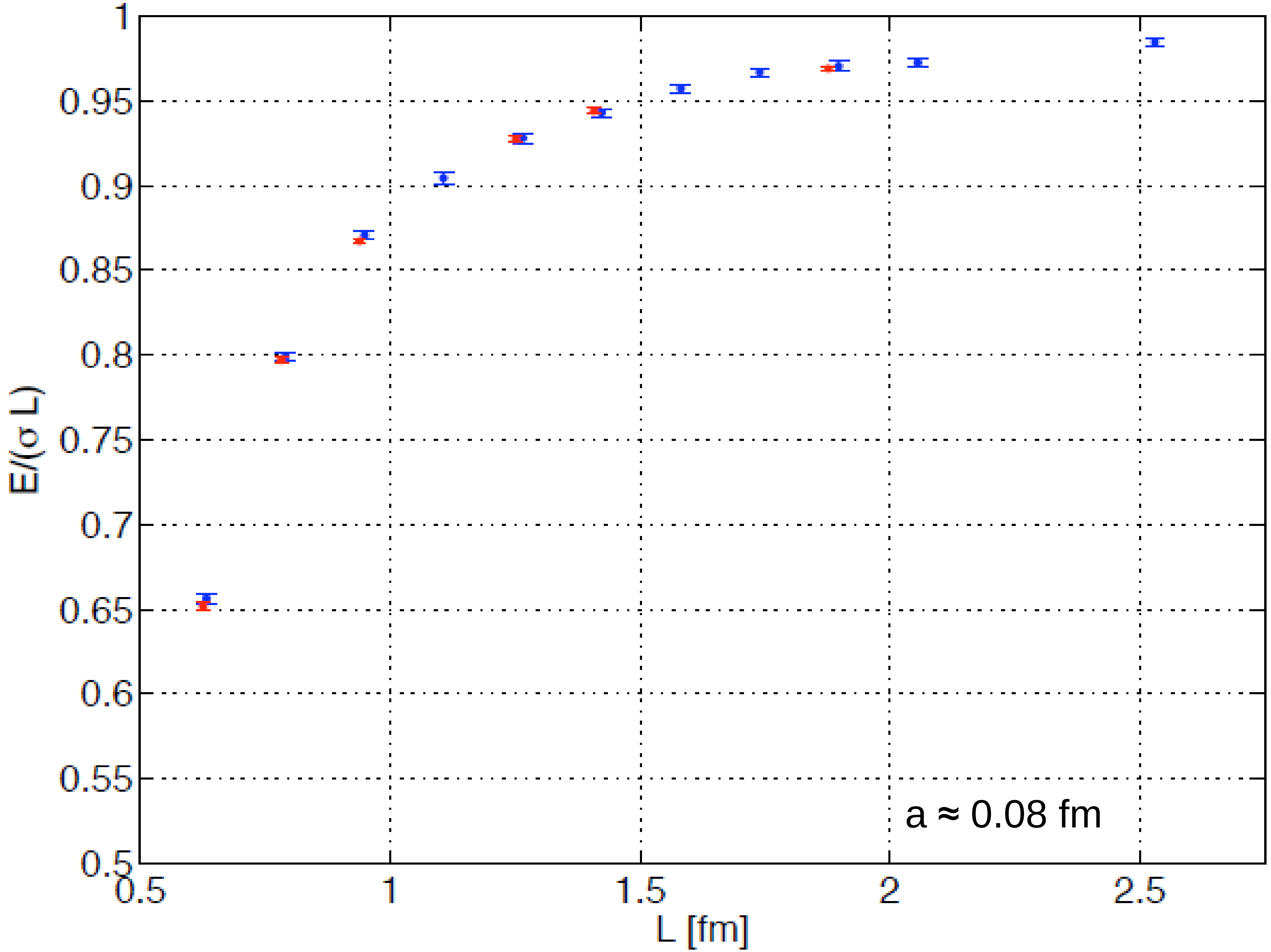
- Since we have *hundreds* of wave functions to span space ≈ <u>98-99.5%</u> purity !

We construct the wave functions of <u>tens</u> of ~pure quantum states ☺

Ground state energy vs flux-tube length (D = 2+1)



(Ground state energy) / (σL) vs flux-tube length (D = 2+1)



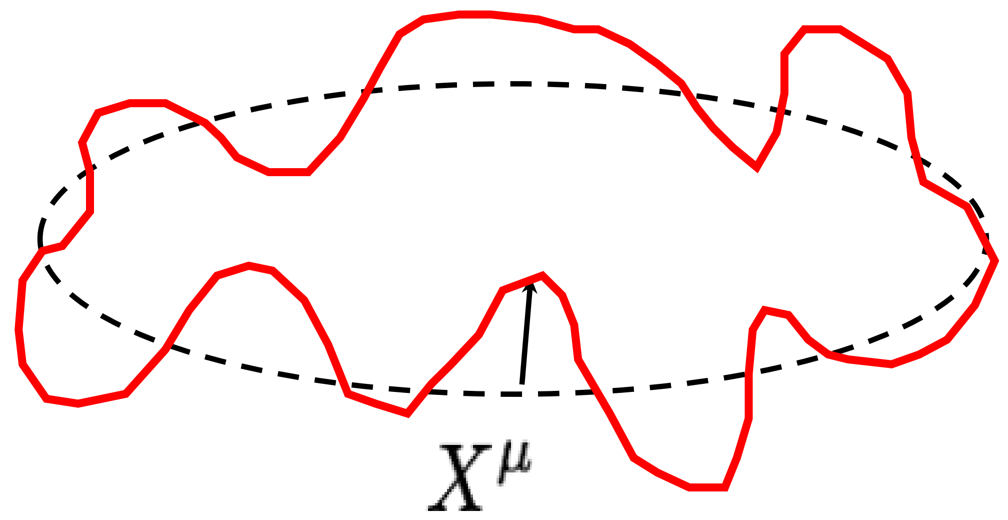
Model predictions for E(L)

All assume that flux-tube is an ***effective*** infinitely thin string:

Circumference = L

L >> width

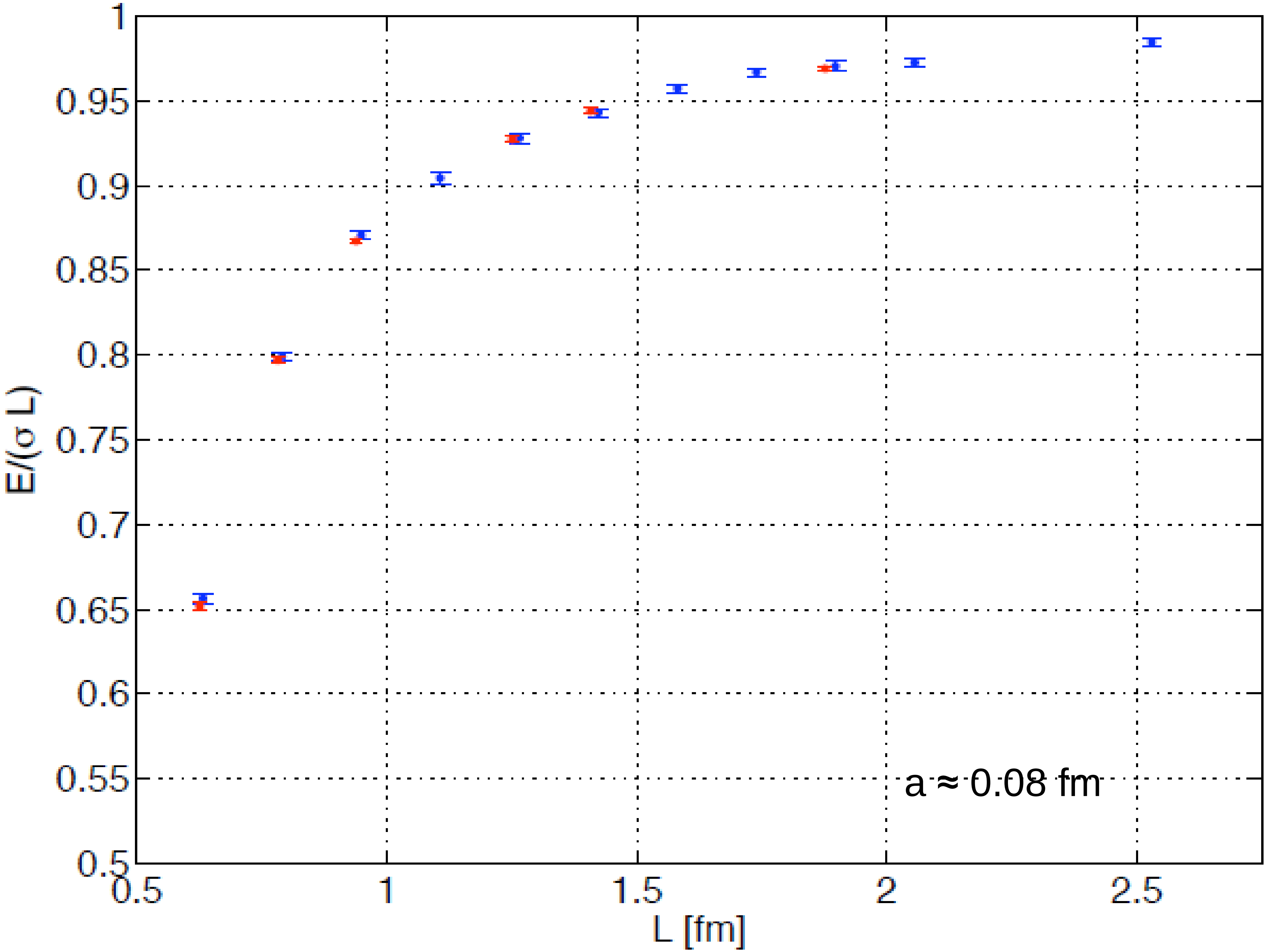
=



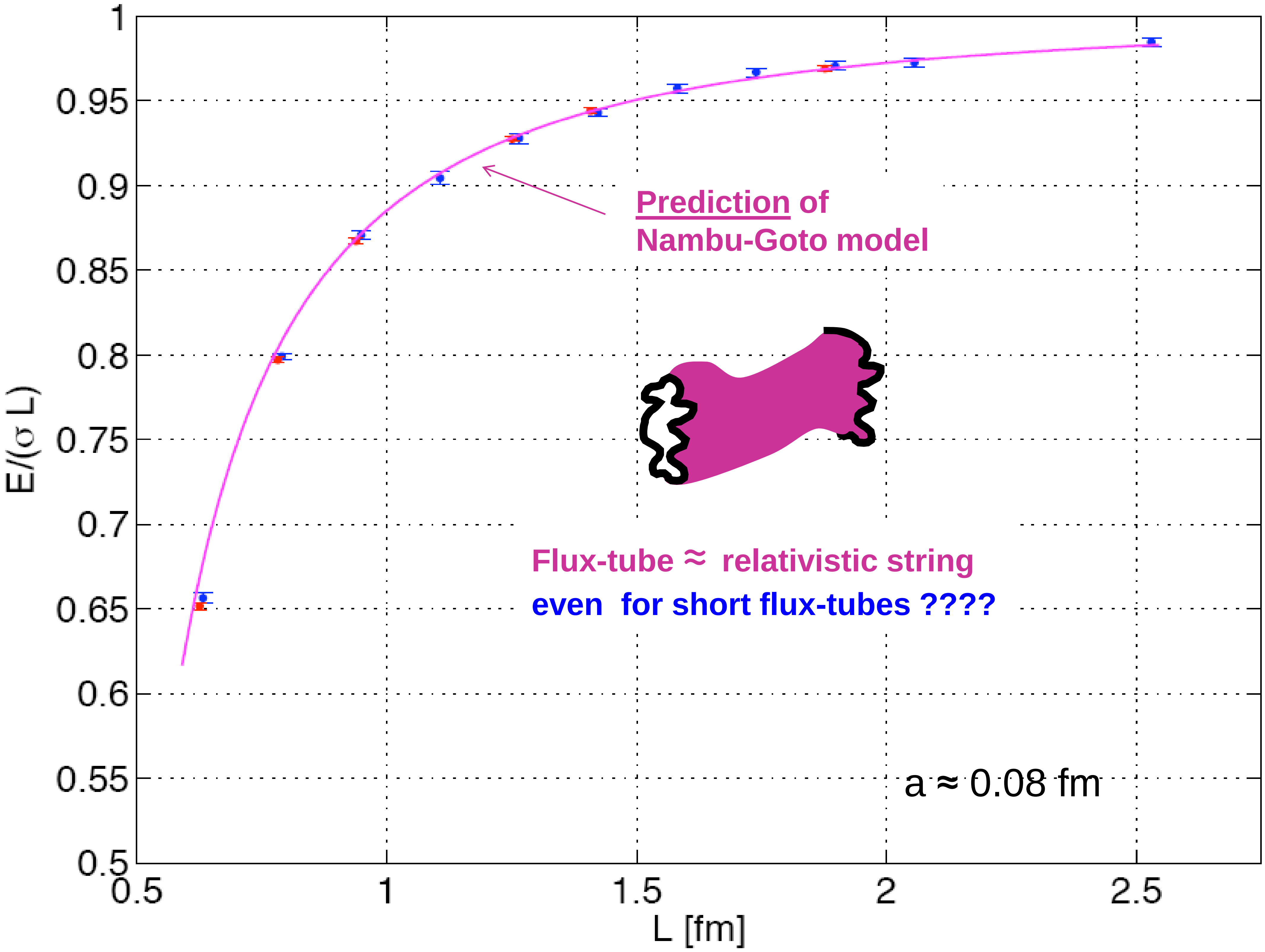
Nambu-Goto Model `70-`71: prediction of E(L) for all L

physics determined by area
the string sweeps while propagating

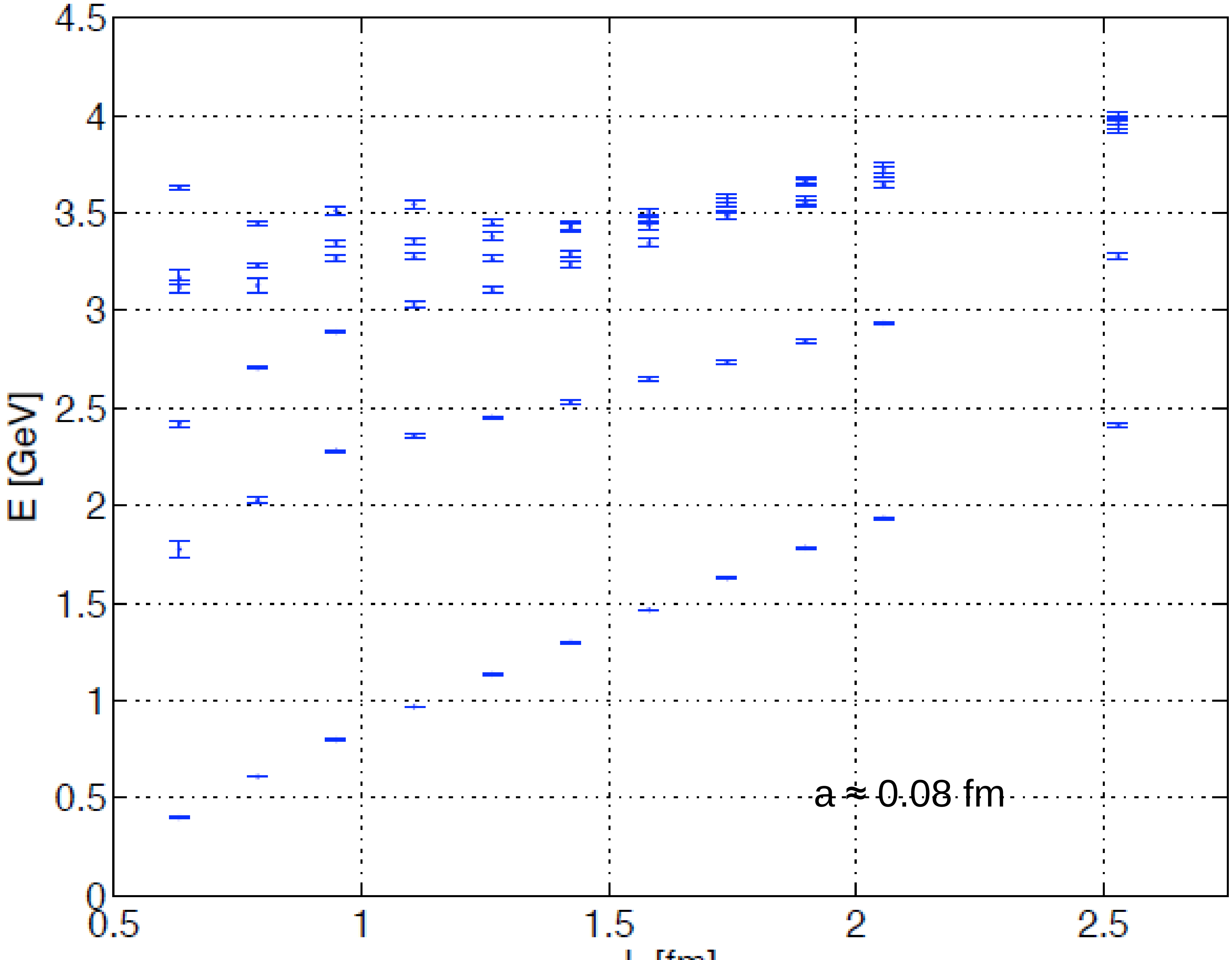
(Ground state energy) / (σL) vs flux-tube length (D = 2+1)



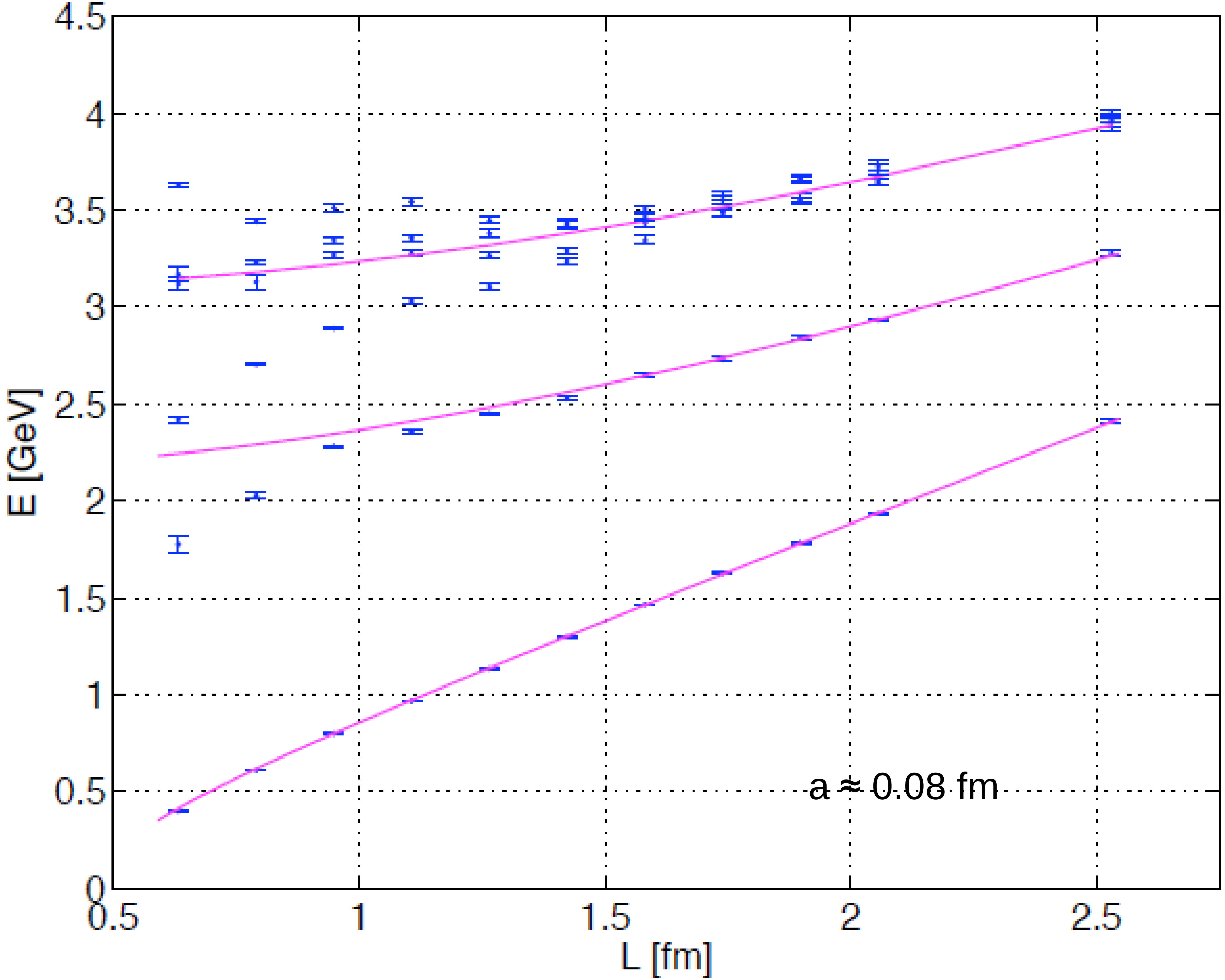
(Ground state energy) / (σL) vs flux-tube length (D = 2+1)



Higher excited states vs flux-tube length D = 2+1

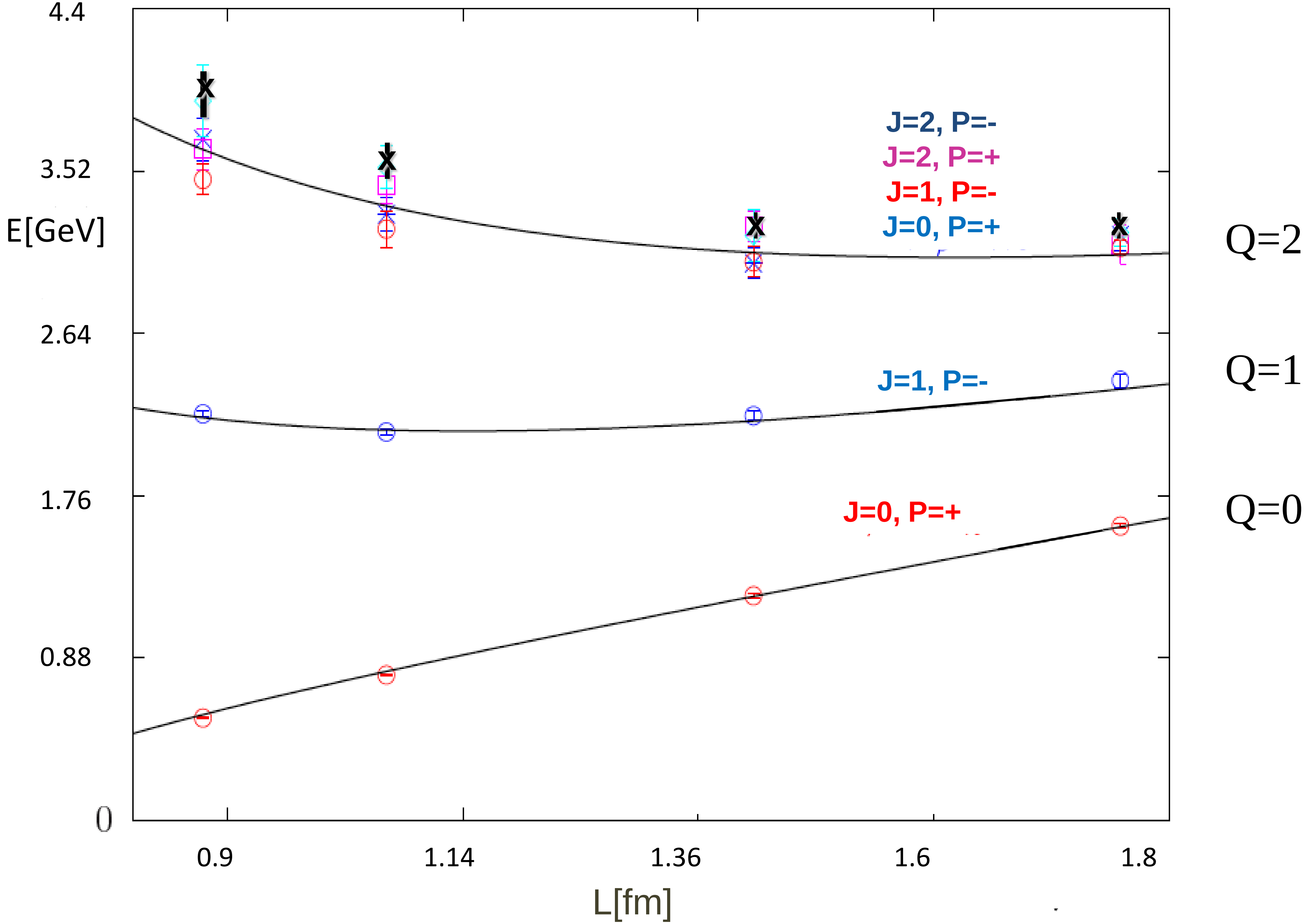


Higher excited states vs flux-tube length   D = 2+1



Ground state: in channels Q=0,1,2, D=3+1

a=0.089fm

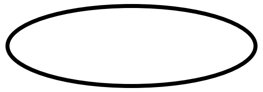


This concludes what I wanted to share with you regarding

Quarks, gluons
&
confining flux-tubes / strings on the lattice

What to take away from this talk

- QCD flux-tubes behave very much like simple strings; true for both

long flux-tubes  Short flux-tubes/blobs 

- The lattice approach is well suited & a technically mature strategy to approach many non-perturbative quantum physics.

Thanks for your time & attention